# "The Environment and Directed Technical Change"

# by Acemoglu et al. (2012)

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Env Reading Group

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# Research Question

- RQ: (i) What determines the direction of technical changes between dirty and clean sectors? (ii) What is the optimal environmental policy with directed technical change?
- Main contribution: the environmental policy implication of the directed technical change theory (Acemoglu, 2002)
- Main Results:
  1. The *market size* effect encourages innovation towards the larger input sector, while the *price effect* directs innovation towards the sector with higher price.
  2. When the two sectors are highly substitutable, immediate and decisive intervention is necessary. These policies need to be in place for only a temporary period.
  3. Optimal environmental regulation should always use both carbon tax to control current emissions, and research subsidies to influence the direction of research.
  4. An environmental disaster is less likely when the dirty sector uses an exhaustible resource.

# Setup

- Economy is populated by one unit of continuum of workers, one unit of continuum of scientists and a continuum of entrepreneurs.
- All households have preferences

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \tag{1}$$

$S_t \in [0, \overline{S}]$: the quality of the environment with $\overline{S}$ the quality without any human pollution.

- Assumption: When $S$ reaches $\overline{S}$, the value of the marginal increase in environmental quality is small:

$$\frac{\partial u(C, \overline{S})}{\partial S} = 0$$

# Setup

- Final goods production:

$$Y_t = \left( Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

  $\epsilon$: elasticity of substitution between two sectors. $\epsilon > 1$, substitutes; $\epsilon < 1$, complements.

- Intermediate goods production:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di, \quad j \in \{c, d\} \tag{3}$$

  $A_{jit}$: the quality of machine of type $i$ used in sector $j$;

  $x_{jit}$: the quantity of machine of type $i$ used in sector $j$;

  $L_{jt}$: the quantity of labor used in sector $j$;

# Setup

- Producing one unit of machine costs $\psi$ units of final goods. $\psi = \alpha^2$.
- At the beginning of every period, each scientist decides whether to direct her research to clean or dirty technology;
- She is randomly allocated to one machine and is successful in innovation with probability $\eta_j \in (0, 1)$, where innovation increases the quality of a machine by a factor $1 + \gamma$;
- A successful scientist obtained a one-period patent and becomes the entrepreneur for the current period in the production of machine $i$;
- When innovation is not successful, monopoly rights are allocated randomly to an entrepreneur drawn from the pool of potential entrepreneurs, who then uses the old technology.

# Setup

- The evolution of sector productivity:

$$\begin{aligned} A_{jt} &= \int_0^1 A_{jit} di \\ &= \int_0^1 \left\{ s_{jt} \left[ \eta_j (1+\gamma) A_{jit-1} + (1-\eta_j) A_{jit-1} \right] + (1-s_{jt}) A_{jit-1} \right\} di \\ &= (1+\gamma\eta_j s_{jt}) A_{jit-1} \end{aligned} \tag{4}$$

$s_{jt}$ is the mass of scientists working on machines in sector $j \in \{c, d\}$.

- The quality of environment evolves

$$S_{t+1} = \min \left\{ \max \left[ -\xi Y_{dt} + (1+\delta) S_t \right] \right\} \tag{5}$$

- **Definition**: An environmental disaster occurs if $S_t = 0$ for some $t < \infty$.

# Laissez-Faire Equilibrium

- Final goods producers' optimization:

$$\max_{Y_{dt}, Y_{ct}} \quad Y_t - p_{dt} Y_{dt} - p_{ct} Y_{ct}$$

$$\Rightarrow p_{jt} = \left( \frac{Y_{jt}}{Y_t} \right)^{-1/\epsilon} \tag{6}$$

- Input producers' optimization:

$$\max_{x_{jit}, L_{jt}} \quad p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di - w_t L_{jt} - \int_0^1 p_{jit} x_{jit} di$$

$$\Rightarrow x_{jit} = \left( \frac{\alpha p_{jt}}{p_{jit}} \right)^{1/(1-\alpha)} A_{jit} L_{jit} \tag{7}$$

$$\Rightarrow w_t = (1-\alpha) p_{jit} L_{jt}^{-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di \tag{8}$$

# Laissez-Faire Equilibrium

- Machine producers' optimization:

$$\max_{p_{jit}, x_{jit}} \quad (p_{jit} - \psi) x_{jit}$$

$$\text{s.t.} \quad x_{jit} = \left( \frac{\alpha p_{jt}}{p_{jit}} \right)^{1/(1-\alpha)} A_{jit} L_{jt}$$

$$\Rightarrow p_{jit} = \frac{\psi}{\alpha} = \alpha \tag{9}$$

- The equilibrium profit of machine producers with technology $A_{jit}$ is

$$\pi_{jit} = (1-\alpha)\alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jit} \tag{10}$$

- The expected profits for a scientist engaged in sector $j$ is

$$\begin{aligned} \Pi_{jt} &= \eta_j \int_0^1 (1-\alpha)\alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt} (1+\gamma) A_{jit-1} di \\ &= \eta_j (1-\gamma)(1-\alpha)\alpha p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jt-1} \end{aligned} \tag{11}$$

# Directed Technical Change

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \underbrace{\left(\frac{p_{ct}}{p_{dt}}\right)^{1/(1-\alpha)}}_{\text{price effect}} \underbrace{\left(\frac{L_{ct}}{L_{dt}}\right)}_{\text{market size effect}} \underbrace{\left(\frac{A_{ct-1}}{A_{dt-1}}\right)}_{\text{productivity effect}} \tag{12}$$

- Productivity effect pushes innovation towards the sector with higher productivity;
- Price effect encourages innovation towards the sector with higher prices;
- Market size effect encourages innovation in the sector with greater market for machines.

# Laissez-Faire Equilibrium

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)}, \quad \frac{L_{ct}}{L_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi}, \quad \varphi \equiv (1-\alpha)(1-\epsilon) \underbrace{<}_{\epsilon>1} 0$$

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-(1+\varphi)}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi} \tag{13}$$

**Lemma**

*Under laissez-faire, it is an equilibrium for innovation at time t to occur*

- *only in the clean sector* $\Rightarrow \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi} > \frac{\eta_d}{\eta_c}(1+\gamma\eta_c)^{1+\varphi}$
- *only in the dirty sector* $\Rightarrow \left(\frac{A_{dt-1}}{A_{ct-1}}\right)^{-\varphi} > \frac{\eta_c}{\eta_d}(1+\gamma\eta_d)^{1+\varphi}$
- *in both sectors* $\Rightarrow \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi} = \frac{\eta_d}{\eta_c}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{(1+\varphi)}$

# Environmental Disaster

- Assumption: $\frac{A_{c0}}{A_{d0}} < \min\left\{(1+\gamma\eta_c)^{-\frac{\varphi+1}{\varphi}}\left(\frac{\eta_c}{\eta_d}\right)^{1/\varphi}, (1+\gamma\eta_d)^{\frac{1+\varphi}{\varphi}}\left(\frac{\eta_c}{\eta_d}\right)^{1/\varphi}\right\}$

**Proposition**

*Suppose that Assumption 1 holds. Then there exists a unique laissez-faire equilibrium where innovation always occurs in the dirty sector only, and the long-run growth rate of dirty input production is $\gamma\eta_d$. The laissez-faire equilibrium always leads to an environmental disaster.*

- Innovation starts in the dirty sector $\Rightarrow$ only $A_dt$ grows, which further increases the technical gap.
- Dirty input production grows at the same rate as $A_{dt}$ in the long run

$$Y_{dt} = \frac{A_{dt}}{\left(1+\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right)^{\frac{\alpha+\varphi}{\varphi}}} \tag{14}$$

# Research Subsidy

- Suppose that the government can subsidize scientists to work in the clean sector with a rate $q_t$, then the expected profit from innovation in the clean sector is

$$\Pi_{ct} = (1 + q_t)\eta_c(1 + \gamma)(1 - \alpha)\alpha p_{ct}^{1/(1-\alpha)} L_{ct} A_{ct-1}$$

- A sufficiently high subsidy to clean research can redirect innovation towards the clean sector;
- A temporary subsidy is sufficient to redirect all research to the clean sector: When the ratio $A_{ct}/A_{dt}$ becomes sufficiently high, it will be profitable for scientists to direct their research to the clean sector even without the subsidy.

# Research Subsidy

- A temporary subsidy is sufficient to avoid an environmental disaster when two inputs are strong substitutes ($\epsilon > 1/(1-\alpha)$): $\alpha + \varphi < 0$, $Y_{dt}$ will not grow in the long run;
- A temporary subsidy cannot prevent an environmental disaster when two inputs are weak substitutes ($\epsilon \in (1, 1/(1-\alpha))$): Even though $A_{dt}$ is constant, $Y_{dt}$ grows at the rate $(1+\gamma\eta_c)^{\alpha+\varphi} - 1$
- As the average quality of clean machines increases, workers are reallocated towards the clean sector: $\frac{L_{ct}}{L_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi}$.
- The increase of the relative price of the dirty input encourages production of the dirty input.

# Socially Optimal Allocation

- There are three kinds of externality:
  1. the environmental externality exerted by dirty input producers;
  2. the knowledge externalities from R&D: scientists do not internalize the effects of their research on productivity in the future;
  3. monopoly distortion;

**Proposition**

*The socially optimal allocation can be implemented using a tax on dirty input (a carbon tax), a subsidy to clean innovation, and a subsidy for the use of all machines.*

- The underutilization of machines due to monopoly is corrected by a subsidy for machines;
- The optimal carbon tax is equal to the social cost of carbon: the marginal cost of reducing the production of dirty input by one unit must be equal to the resulting marginal benefit in terms of higher environmental quality;
- The subsidy to clean sector allocates scientists to the sector with the higher social gain from innovation.

# Socially Optimal Allocation

- By reducing production in the dirty sector, the carbon tax also discourages innovation in that sector.
- So why is a research subsidy to clean sector still necessary?
- Because using only the carbon tax to deal with two externalities will necessitate a higher carbon tax, distorting current production and reducing current consumption.

# Structure of Optimal Environmental Regulation

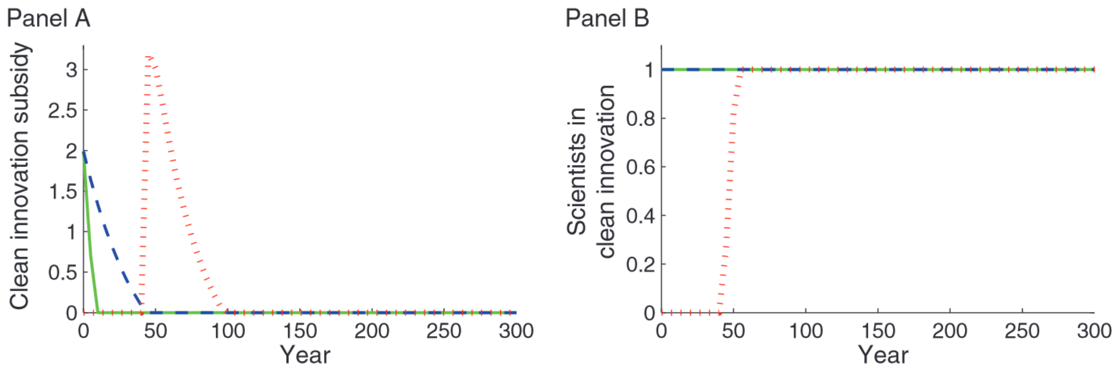
**Proposition**

*Suppose that $\epsilon > 1$ and $\rho$ is sufficiently small. Then the optimal subsidy in the clean sector is temporary. Moreover, if $\epsilon > 1/(1-\alpha)$, then the optimal carbon tax is temporary.*

- An optimal policy requires avoiding a disaster.
- When the discount rate is sufficiently low, it is optimal to have positive long-run growth,which can be achieved by technical change in the production of the clean input, without growth in the production of the dirty input.
- Failing to allocate all research to clean innovation would reduce intertemporal welfare.

# Results

- Green: $\epsilon = 10, \rho = 0.015$; Blue: $\epsilon = 3, \rho = 0.001$; Red: $\epsilon = 3, \rho = 0.015$



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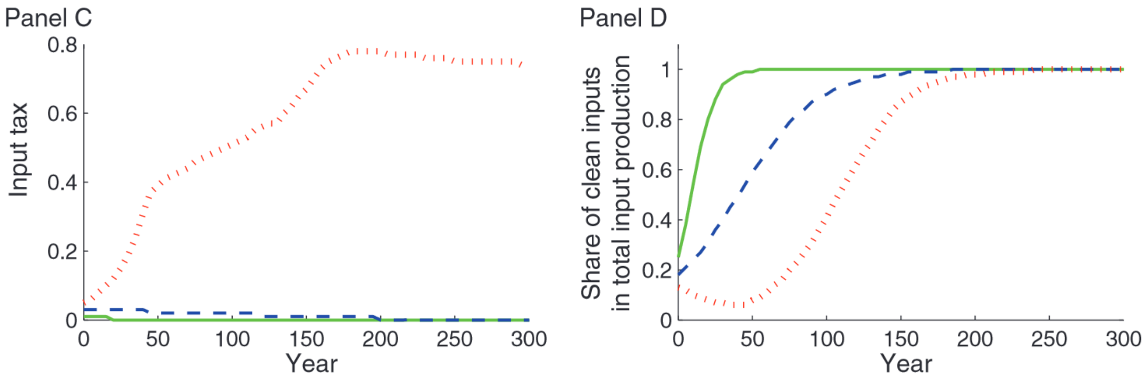

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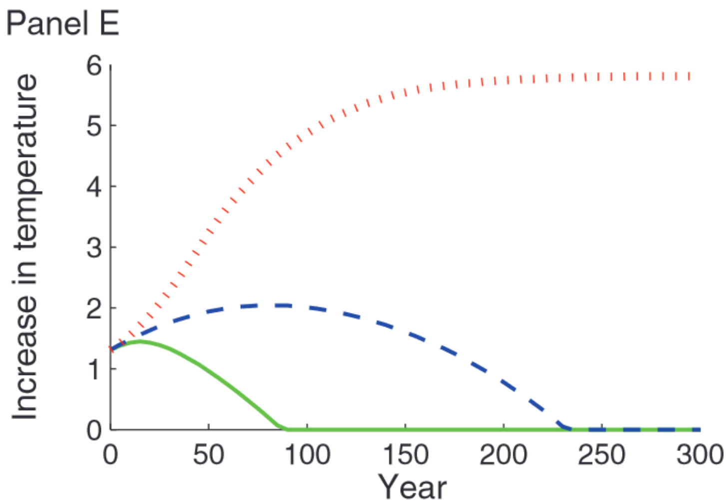

# Results

TABLE 2—WELFARE COSTS OF RELYING SOLELY ON CARBON TAX AS A FUNCTION OF THE ELASTICITY OF SUBSTITUTION AND THE DISCOUNT RATE

| Elasticity of substitution $\varepsilon$ | 10 | | 3 | |
|---|---|---|---|---|
| Discount rate $\rho$ | 0.001 | 0.015 | 0.001 | 0.015 |
| Welfare cost | 1.02 | 1.66 | 1.92 | 3.15 |

*Note:* Percentage reductions in consumption relative to immediate intervention.

- Welfare loss is smaller when $\epsilon$ is higher: a relatively small carbon tax can redirect R&D towards clean technologies;
- Welfare loss is larger when $\rho$ is higher: a higher discount rate puts greater weight on earlier periods.

# References

Acemoglu, D. (2002). Directed technical change. *The review of economic studies*, 69(4):781–809.

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012). The environment and directed technical change. *American economic review*, 102(1):131–166.