"Carbon Taxes and Climate Commitment with Non-constant Time Preference" by Iverson and Karp (2021)

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Introduction

- The value of commitment in climate regulation when households have inconsistent time preference.
- Time inconsistency: the discount rate future agents will use is different from the rates the current agent would like them to use

$$V_t = \sum_{\tau=0}^{\infty} \lambda_\tau u_{t+\tau} \tag{1}$$

- The paper finds that quantitatively commitment over realistic horizons is worth little.

Literature Contribution

- Golosov et al. (2014):
  1. constant time preference;
  2. log-linear model. Analytical solution: carbon tax in each period is proportional to output: $\tau_t = aY_t$.
- Gerlagh and Liski (2018):
  1. non-constant time preference;
  2. log-linear model (analytical solution);
  3. planner control both carbon emissions and investment;
- Iverson and Karp (2021):
  1. general functional forms.
  2. planner only controls carbon emissions: A novel numerical approach to solve a sequence of fixed point problem

Model

- Household welfare in t is

$$\sum_{\tau=0}^{\infty} \lambda_\tau u(c_{t+\tau}), \quad \lambda_0 = 1, \{\lambda_\tau\}_{\tau=0}^{\infty} \text{ is decreasing} \tag{2}$$

- A representative firm with production function

$$Y_t = F_t(K_t, E_t, S_t) \tag{3}$$

$$r_t(K_t, E_t, S_t) = \frac{\partial F_t(K_t, E_t, S_t)}{\partial K_t}; \tag{4}$$

$$w_t(K_t, E_t, S_t) = Y_t - r_t K_t \tag{5}$$

- The climate state evolves according to

$$S_{t+1} = f(S_t, E_t) \tag{6}$$

- Resource constraint:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \tag{7}$$

Savings

- The household t's welfare is

$$u(r_t k_t + w_t + (1-\delta)k_t - k_{t+1}) + \sum_{j=1}^{\infty} u(r_{t+j}k_{t+j} + w_{t+j} + (1-\delta)k_{t+j} - k_{t+j+1}) \quad (8)$$

  With NCTP, household t can only choose current savings, k_{t+1}.

- In Markov perfect equilibrium, they have a rational expectation of his succors' saving policy. They try to manipulate future savings by choosing k_{t+1}.
- When household t makes decision, he needs to know
  1. his current capital, k_t;
  2. current prices which are determined by (K_t, S_t, E_t);
  3. succors' savings, $k_{t+j}, j > 1$;
  4. future prices determined by $(K_{t+j}, S_{t+j}, E_{t+j}), j \geq 1$
- Individual saving rule: $g_t(k_t, K_t, S_t; \{E_s\}_{s=t}^{\infty})$; Aggregate saving rule: $G_t(K_t, S_t; \{E_s\}_{s=t}^{\infty})$.

Savings

- Household t choose current savings, k_{t+1}, takes as given its successors' saving rules, $g_s(k_s, K_s, S_s; \{E_m\}_{m=s}^{\infty})$, $s > t$, the current and future aggregate savings rules, $G_s(K_s, S_s; \{E_m\}_{m=s}^{\infty})$, $s \geq t$, to maximize

$$u(r_t k_t + w_t + (1-\delta)k_t - k_{t+1}) + \tilde{V}_{t+1}(k_{t+1}, K_{t+1}, S_{t+1}) \tag{9}$$

$$\text{s.t} \quad k_{t+j} = g_{t+j}(k_{t+j-1}, K_{t+j-1}, S_{t+j-1}; \{E_s\}_{s=t+j}^{\infty}) \tag{10}$$

$$K_{t+j} = G_{t+j}(K_{t+j-1}, S_{t+j-1}; \{E_s\}_{s=t+j-1}^{\infty}) \tag{11}$$

where

$$\begin{aligned} &\tilde{V}_{t+1}(k_{t+1}, K_{t+1}, S_{t+1}) \\ &= \sum_{j=1}^{\infty} \lambda_j u(r_{t+j}k_{t+j} + w_{t+j} + (1-\delta)k_{t+j} - g_{t+j}(k_{t+j}, K_{t+j}, S_{t+j}, \{E_s\}_{s=t+j}^{\infty})) \end{aligned} \tag{12}$$

Saving

- The household t's welfare is

$$V_t(k_t, K_t, S_t; \{E_s\}_{s=t}^{\infty}) = u(r_t k_t + w_t + (1-\delta)k_t - g_t(k_t, K_t, S_t; \{E_s\}_{s=t}^{\infty})) + \tilde{V}_{t+1}(g_t(k_t, K_t, S_t; \{E_s\}_{s=t}^{\infty}), G_t(K_t, S_t; \{E_s\}_{s=t}^{\infty}), S_{t+1}) \tag{13}$$

- In equilibrium, $k_t = K_t$. The equilibrium welfare of household t

$$V_t^e(K_t, S_t; \{E_s\}_{s=t}^{\infty}) = V_t(K_t, K_t, S_t; \{E_s\}_{s=t}^{\infty}) \tag{14}$$

Climate Policy

- In each period, a planner chooses policy to maximize equilibrium welfare for the contemporaneous representative household, taking future climate policy rules as give.
- Commitment device: A planner in t with a j-period commitment device chooses climate policy, E_s for periods $s = t, \cdots, t+j-1$.
- The planner t solves

$$\max_{\{E_s\}_{s=t}^{t+j-1}} \quad V_t^e(K_t, S_t; \{E_s\}_{s=t}^{t+j-1}, \{E_s(K_s, S_s)\}_{s=t+j}^{\infty}) \tag{15}$$

A Markov perfect equilibrium satisfies the following for all t:

- Prices satisfy equation (4) and (5);
- The physical constraints in (3), (6) and (7).
- Individual maximization;
- Individual and aggregate savings are consistent:

$$g_t(K_t, K_t, S_t) = G_t(K_t, S_t) \tag{16}$$

[Fixed point problem: Individual choose $g_t(k_t, K_t, S_t)$ based on perceived aggregate savings $G_t^p \Rightarrow$ individual savings determine aggregate savings $G_t^p \to G_t$]

- Planners choose optimal climate policy;

Log-Linear Model

The log-linear model contains

1. The utility function is logarithmic: $u(C) = \ln(C)$;
2. Full depreciation: $\delta = 1$;
3. Output is Cobb-Douglas in capital with multiplicative climate damages:

$$Y_t = \exp(-\gamma(S_t - \overline{S}))K_t^{\alpha}A_t(E_t)$$

4. S is linear in prior emissions:

$$S_t - \overline{S} = \sum_{j=0}^{t+H}(1 - d_j)E_{t-j}$$

Equilibrium Without Commitment

- The aggregate saving rule is

$$K_{t+1} = sY_t,\ s = \frac{\alpha\rho}{1+\rho},\ \rho = \sum_{t=1}^{\infty} \lambda_t \tag{17}$$

- The emissions taxes are

$$\tau_t = \left[\sum_{k=0}^{\infty} \gamma(1-d_k)\lambda_k \Gamma_k\right] Y_t,\ \Gamma_k = \frac{\sum_{m=0}^{\infty} \alpha^m \lambda_{k,k+m}}{\sum_{n=0}^{\infty} \alpha^n \lambda_n} \tag{18}$$

$$\lambda_{k,k+m} = \frac{\partial V_t / \partial u_{t+k+m}}{\partial V_t / \partial u_{t+k}} = \frac{\lambda_{k+m}}{\lambda_k} \tag{19}$$

Intuition of Emissions Taxes

- Social cost of carbon: the stream of future damages (measured in units of consumption goods) due to an extra unit of emissions, weighted by the appropriate marginal rates of substitution (MRS).
- Climate damage from an extra unit of emissions in t reduce Y_{t+k} by

$$-\frac{\partial Y_{t+k}}{\partial S_{t+k}} \frac{\partial S_{t+k}}{\partial E_t} = \gamma(1-d_k)Y_{t+k} \quad (20)$$

- MRS: the exchange rate of output in $t+k$ to the output in t:

$$V_t = \sum_{v_0}^{\infty} \lambda_v \ln(C_{t+v}) \quad (21)$$

$$MRS_{t,t+k} = \frac{\partial V_t / \partial Y_{t+k}}{\partial V_t / \partial Y_t} \quad (22)$$

- Constant saving rate: $Y_t \uparrow 1\% \rightarrow K_{t+1} \uparrow 1\% \rightarrow Y_{t+1} \uparrow \alpha\% \rightarrow K_{t+2} \uparrow \alpha\% \rightarrow Y_{t+2} \uparrow \alpha^2\%$ $\cdots Y_{t+v} \uparrow \alpha^v\%$

Intuition of Emissions Taxes

$$\frac{\partial V_t}{\partial Y_t} = \sum_{v=0}^{\infty} \lambda_t \frac{\partial \ln(C_{t+v})}{\partial \ln(Y_t)} \frac{\partial \ln(Y_t)}{\partial Y_t} = \sum_{v=0}^{\infty} \lambda_v \alpha^v \frac{1}{Y_t} \tag{23}$$

$$\frac{\partial V_t}{\partial Y_{t+k}} = \sum_{v=0}^{\infty} \lambda_{k+v} \alpha^v \frac{1}{Y_{t+k}} \tag{24}$$

$$MRS_{t,t+k} = \frac{\sum_{v=0}^{\infty} \lambda_{k+v} \alpha^v}{\sum_{v=0}^{\infty} \lambda_v \alpha^v} \frac{Y_t}{Y_{t+k}} = \lambda_k \Gamma_k \frac{Y_t}{Y_{t+k}} \tag{25}$$

- The climate damage in $t+k$ from an extra unit of emissions in t measured in the unit of t output is

$$MRS_{k,t+k} \frac{\partial Y_{t+k}}{\partial S_{t+k}} \frac{\partial S_{t+k}}{\partial E_t} = \gamma(1-d_k)\lambda_k \Gamma_k Y_t \tag{26}$$

Two Decoupling

- Intra-temporal decoupling: aggregate saving rate and equilibrium emissions tax policy (expressed as a fraction of income);
- Inter-temporal decoupling: Emission tax policy in an period is independent of climate policy in all other periods.

Equilibrium With Commitment

- The planner in 0 chooses policy for j future periods;
- Equivalence: At each period $v < j$, a "pseudo-planner" without commitment uses a sequence of time preference that the initial planner would want them to use (consistent preference: $\{\lambda_{v,v+m}\}_{m=0}^{\infty}$
- Reason: The planner at 0 use weights λ_{v+m} for periods $v+m$. (time inconsistent planner use λ_m)
- The weights are normalized by λ_v to keep the first period of all pseudo planner is 1.

Equilibrium With Commitment

- In equilibrium, carbon taxes within commitment interval are

$$\tau_v = \left[\sum_{k=0}^{\infty} \gamma(1 - d_k)\lambda_{v+k}\tilde{\Gamma}_k^v\right] Y_v \tag{27}$$

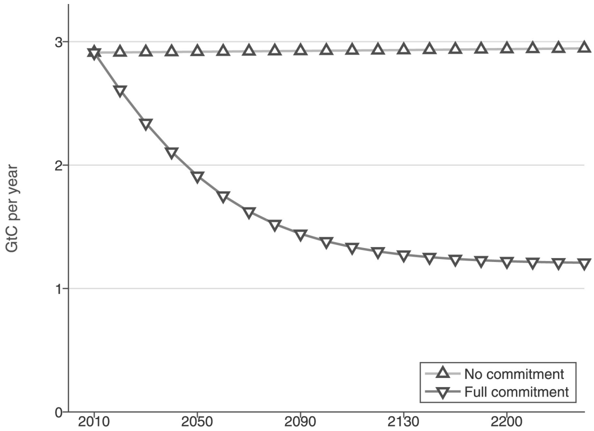
$$\tilde{\Gamma}_k^v = \frac{\sum_{m=0}^{\infty} \alpha^m \lambda_{v+k,v+k+m}}{\sum_{n=0}^{\infty} \alpha^n \lambda_{v+n}} \tag{28}$$

- Carbon taxes without commitment interval are the same as the equilibrium without commitment.

Value of Commitment

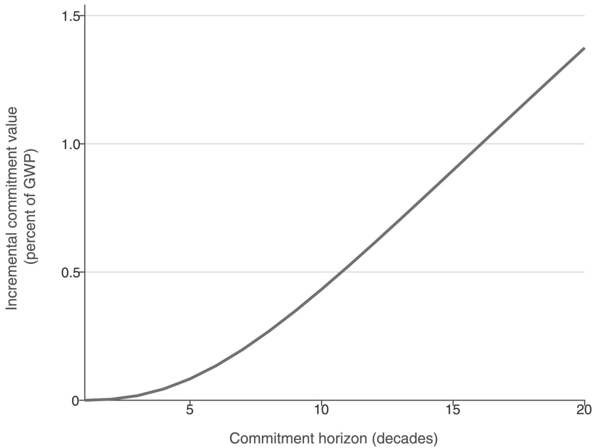
- The value of commitment: the increment in first-period consumption needed in the scenario without commitment to raise welfare to the level with commitment.
- Calculation: Divide the difference in welfare, with and without commitment, by the initial period marginal utility of consumption in the no-commitment scenario;

Value of Full Commitment



- Absent commitment, future generations do less to combat climate change than the initial generation would like.
- The permanent commitment device is worth 16 trillion USD (2.3% of the initial decade's Gross World Product)

Value of Partial Commitment



- The curve is initially convex, eventually concave and converge to a max;
- A small amount of commitment provides only a small welfare increase.
- 50-year commitment window is worth about 0.1% of the first decade's GWP.

Value of Partial Commitment

- Convex (Concave): One extra period of commitment becomes more important (unimportant);
- The offsetting forces: decision-conflict effect (convex) VS present-value effect (concave)
- Decision-conflict effect: the discount rate that the agent in t uses for period $t+N$ (λ_N) is different from the agent in $t+N$ (1).
- Since λ_N is decreasing in N, the degree of conflict between agents grows in $N \Rightarrow$ the added decade of commitment resolves a greater conflict
- Present-value effect: More serious conflict occur further in the future $\Rightarrow$ discount more heavily

References

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