

October 13, 2023

Research Question

- Research Questions:
 - ① In the context of a micro-founded quantitative model, can reasonable policies secure a transitional to clean technology?
 - ② Is there a important role for significant research subsidies conditional on optimally chose carbon taxes?
 - ③ How rapidly should the transition to clean technology take place under optimal policies?
- To answer the three questions, the paper builds a tractable endogenous growth model.
- Match the model with micro-data and used the calibrated model for the analysis of optimal policy and for range of counterfactual policy experiments.

Main Results

- Both the carbon taxes and research subsidies to clean innovation are important.
- The research subsidy is initially more aggressive (to redirect technological change from dirty to clean technology) and then declines over time, while optimal carbon tax is back-loaded.
- The cost of relying only on the carbon tax is equivalent to 1.9% permanent drop in consumption and delaying optimal policies by 50 years has a welfare cost of 1.7%.

Model Overview

- The model is composed of two parts:
 - ① Macro part (negligible): Households and Final Goods Production; Environment Dynamics; Resource Extraction;
 - ② Micro part: Intermediate Goods Production; Directed Innovation;
- Final goods, used for consumption, is produced by a continuum of intermediate goods.
- Each intermediate good is produced by either clean and dirty technology. The labor productivity of producing each good is represented by a quality ladder; [Draw]
- Firms (who might produce several intermediate goods) also decide whether to conduct research to improve clean or dirty technologies.
- Successful innovation has two types: incremental innovation and breakthrough innovation;
- Government has two policy instruments: product taxes and research subsidies.

Preference and Final Goods

- The representative household has preference:

$$U_0 = \int_0^{\infty} e^{-\rho t} \ln(C_t) dt \quad (1)$$

- The supply of unskilled workers and skilled workers are 1 and L^s . Unskilled workers are employed during production and resource extraction while skilled workers are employed in R&D activities.
- Final goods production technology

$$\ln Y_t = -\gamma(S_t - \bar{S}) + \int_0^1 \ln y_{it} di \quad (2)$$

$$\Rightarrow p_{it} = \left(\frac{y_{it}}{Y_t} \right)^{-1} \quad (3)$$

where S_t is the atmospheric carbon concentration.

Intermediate Goods: Production

- Intermediate goods can be produced either by clean or dirty technology:

$$y_{it} = \begin{cases} q_{it}^c l_{it}, & \text{if it is produced with clean technology} \\ q_{it}^d l_{it}^{1-\nu} e_{it}^\nu = \left(\frac{1-\nu}{\nu} \frac{p_{et}}{w_t^u} \right)^\nu q_{it}^d l_{it}, & \text{if it is produced with dirty technology} \end{cases} \quad (4)$$

- The tax-adjusted marginal cost of product line i is

$$MC_{it} = \begin{cases} \frac{(1+\tau_t^c)w_t^u}{q_{it}^c} = \frac{w_t^u}{\tilde{q}_{it}^c}, & \text{if it is produced with clean technology} \\ \frac{(1+\tau_t^d)\tilde{P}_{et}w_t^u}{q_{it}^d} = \frac{w_t^u}{\tilde{q}_{it}^d}, & \text{if it is produced with dirty technology} \end{cases} \quad (5)$$

where $\tilde{P}_{et} = \left(\frac{1-\nu}{\nu} \frac{p_{et}}{w_t^u} \right)^\nu$

Intermediate Goods: Pricing

- For each product line i , firms compete in price. Firms are heterogeneous in tax-adjusted labor productivity \tilde{q}_{it} . (We will talk about the reason in the next slide)
- Firms with the lowest marginal cost dominate the market of product i :

$$\text{Good } i \text{ is produced } \begin{cases} \text{with clean technology,} & \tilde{q}_{it}^c > \tilde{q}_{it}^d \\ \text{with dirty technology,} & \tilde{q}_{it}^c < \tilde{q}_{it}^d \end{cases} \quad (6)$$

Note: From now on, the quality \tilde{q}_{it}^j represents the leading-edge technology (the highest labor productivity) in sector j

- If the leading-edge technology for good i at time t is \tilde{q}_{it}^j , then the market price of good i is

$$p_{it} = \min \left\{ \frac{w_t^u \lambda}{\tilde{q}_{it}^j}, \frac{w_t^u}{\tilde{q}_{it}^{-j}} \right\}, \quad \Rightarrow \quad y_{it} = \max \left\{ \frac{\tilde{q}_{it}^j}{w_t^u \lambda}, \frac{\tilde{q}_{it}^{-j}}{w_t^u} \right\} Y_t \quad (7)$$

Note: The firm will set the price equal to the second lowest marginal cost. Because of the unit demand elasticity and price competition.

Intermediate Goods: Innovation Process

- Labor productivity evolves as a result of innovation.
- Labor productivity follows a quality ladder: each rung in the quality ladder corresponds to a proportional improvement $\lambda > 1$. Hence,

$$q_{it}^j = \lambda^{n_{it}^j}$$

where n_{it}^j is the effective number of steps that this technology has taken.

- If research directed to sector $j \in \{c, d\}$, successful innovation leads to two types of innovation in technology j **in a randomly chosen intermediate**.
- If an innovation of technology j happens in goods i between t and $t + \Delta t$:

$$\begin{cases} \text{Incremental innovation (with probability } 1 - \alpha): q_{it+\Delta t}^j = \lambda q_{it}^j \\ \text{Breakthrough innovation (with probability } \alpha): q_{it+\Delta t}^j = \lambda \left(\mathbb{I}_{\{n_{it}^j < n_{it}^{-j}\}} q_{it}^{-j} + \mathbb{I}_{\{n_{it}^j \geq n_{it}^{-j}\}} q_{it}^j \right) \end{cases}$$

Intermediate Goods: Innovation Process

- Let z_t^j denote the aggregate innovation rate in technology j . (It is endogenously determined by firms R& D efforts, which are shown in the following slides)
- Suppose $n_{it}^c > n_{it}^d$, then the evolution of n_{it}^d and n_{it}^c are

$$n_{it+\Delta t} = \begin{cases} n_{it}^c + 1, & \text{with probability } z_t^c \Delta t \\ n_{it}, & \text{with probability } 1 - z_t^c \Delta t \end{cases}$$

$$n_{it+\Delta t} = \begin{cases} n_{it}^d + 1, & \text{with probability } z_t^d \alpha \Delta t \\ n_{it}^c + 1, & \text{with probability } z_t^d (1 - \alpha) \Delta t \\ n_{it}^d, & \text{with probability } 1 - z_t^d \Delta t \end{cases}$$

Intermediate Goods: Innovation Process

- Let $n_{it} \equiv n_{it}^d - n_{it}^c$ denote the technology gap between dirty and clean technology in product line i . The evolution of n_{it} is

$$n_{it+\Delta t} = \begin{cases} n_{it} + 1, & \text{with probability } z_t^d(1 - \alpha)\Delta t + o(\Delta t) \\ n_{it} - 1, & \text{with probability } z_t^c\Delta t + o(\Delta t) \\ 1, & \text{with probability } z_t^d\alpha\Delta t + o(\Delta t) \\ n_{it} & \text{with probability } 1 - (z_t^d + z_t^c)\Delta t + o(\Delta t) \end{cases}$$

- The relative productivity of dirty to clean technology in product line i can be expressed as

$$\frac{q_{it}^d}{q_{it}^c} = \lambda^{n_{it}}, \quad \Rightarrow \quad \frac{\tilde{q}_{it}^d}{\tilde{q}_{it}^c} = \lambda^{n_{it} - m_t} \quad \text{where} \quad \lambda^{m_t} \equiv \frac{(1 + \tau_t^d)\tilde{P}_{et}}{1 + \tau_t^c}$$

Intermediate Goods: Profits

- Firms holding the leading-edge technology in sector d has profits of good i :

$$\begin{aligned}
 \Pi_{it}^d &= (p_{it} - mc_{it})y_{it} \\
 &= Y_t - \frac{w_t^u}{\tilde{q}_{it}^d} \max \left\{ \frac{\tilde{q}_{it}^d}{\lambda w_t^u}, \frac{\tilde{q}_{it}^c}{w_t^u} \right\} Y_t \\
 &= Y_t \left[1 - \max \left\{ \frac{1}{\lambda}, \frac{1}{\lambda^{n_{it}-m_t}} \right\} \right]
 \end{aligned} \tag{8}$$

- Hence, the profit of a firm holding the leading-edge technology of good i in sector d is a function of the technology gap:

$$\pi_t^d(n) = \begin{cases} 0, & \text{if } n \leq m_t \\ 1 - \frac{1}{\lambda^{n-m_t}}, & \text{if } m_t < n < m_t + 1 \\ 1 - \frac{1}{\lambda}, & \text{if } 1 + m_t \leq n \end{cases} \tag{9}$$

R&D: Incumbents

- Let u_f^j denote the number of intermediates in which firm f has the leading-edge technology within sector j .
- The rate of new innovation in sector j of firm f is

$$X_t^j = \theta (H_t^j)^\eta (u_t^j)^{1-\eta} \quad (10)$$

- The cost of generating innovation rate $x_t^j \equiv X_t^j / u_t^j$ is

$$w_t^s \left(\frac{x_t^j}{\theta} \right)^{\frac{1}{\eta}} u_t^j$$

- Fixed cost: $F_{lt}^f \mu_t^j w_t^s$ where $F_{lt}^f \in [(1-\xi)F_l, (1+\xi)F_l]$. i.i.d draw across firms and over time.
- The total cost of generating x_f^j rate of innovation is

$$w_t^s \mu_t^j \left[\left(\frac{x_t^j}{\theta} \right)^{\frac{1}{\eta}} + F_{lt}^f \right]$$

R&D: Entrants and Subsidies

- Entrants undertake R&D by paying

$$w_t^s \left[\left(\frac{x_{Et}^j}{\theta} \right)^{\frac{1}{\eta}} + F_E \right]$$

where F_E is a constant fixed cost.

- R&D for sector j receives a proportional government subsidy at the rate $s_t^j \in [0, 1]$.
- Hence, the total cost of R&D directed to sector j is

$$C_t^j(x) = \begin{cases} (1 - s_t^j) w_t^s u_t^j \left[\left(\frac{x}{\theta} \right)^{\frac{1}{\eta}} + F_{It} \right] & \text{if the firm has } u_t^j \text{ leading-edge technology in sector } j \\ (1 - s_t^j) w_t^s \left[\left(\frac{x}{\theta} \right)^{\frac{1}{\eta}} + F_E \right] & \text{if the firm is an entrant} \end{cases} \quad (11)$$

Value Functions

- Since there are no technological or product market linkages between the different product lines in which the firm has a lead, a firm's value can be written as a sum of the values of each product line in which the firm has a lead.
- The value of product line i is defined as

$$v^j(n_i, t) = \mathbb{E}_0 \int_0^\infty e^{-\rho t} \pi_t^j(n_i) dt$$

Here n_i is the technology gap ($n_i^j - n_i^{-j}$) within product line i .

Value Functions

- The HJB is:

$$\begin{aligned} \tilde{r}_t^j v_{n_i,t}^j - \dot{v}_{n_i,t}^j &= \pi_{n_i,t}^j - z_t^j v_{n_i,t}^j + z_t^{-j} [1 - \alpha + \mathbb{I}_{\{n_i \leq 0\}} \alpha] (v_{n_i-1,t}^j - v_{n_i,t}^j) \\ &\quad + z_t^{-j} \alpha \mathbb{I}_{\{n_i > 0\}} (v_{-1,t}^j - v_{n_i,t}^j) \\ &\quad + \int \max_{x_t^j \geq 0} \{x_t^j \bar{v}_t^j - (1 - s_t^j) \tilde{w}_t^s [(x_t^j)^{1/\eta} \theta^{-1/\eta} + \mathbb{I}_{\{x_t^j > 0\}} F_{lt}]\} dF_{lt} \end{aligned}$$

- Here, \bar{v}_t^j is the expected per-product value of innovation:

$$\bar{v}_t^c \equiv \sum_{-\infty}^0 \mu_{nt} v_{n-1,t}^c + \sum_1^{\infty} \mu_{nt} [(1 - \alpha) v_{n-1,t}^c + \alpha v_{-1,t}^c] \quad (12)$$

where n is the technology gap between dirty and clean sector and $\mu_{n,t}$ is the fraction of product lines where the dirty lead is equal to n steps at time t .

Value Functions

- The optimal innovation rate per product line is

$$x_t^j = \left[\frac{\eta \theta^{1/\eta} \bar{v}_t^j}{(1 - s_t^j) w_t^s} \right]^{\frac{\eta}{1-\eta}} \quad (13)$$

- **Carbon taxes increase clean research efforts.** Higher carbon tax, higher m_t and lower profit of holding leading-edge technology in clean sector. Then lower expected value from innovation in the clean technology;
- **Innovation is path dependent in this economy.** If an economy is more advanced in clean technology, then the average profit for clean producers ($\sum_{n < m} \mu_{nt} \pi_t^c(n)$) is large. Then the expected value in the clean technology is larger.

Free Entry Condition

- The free-entry condition for technology j is

$$\max_{x_{E,t}^j \geq 0} \{x_{E,t}^j \bar{v}_t^j - (1 - s_t^j) w_t^s [(x_{E,t}^j)^{1/\eta} \theta^{-1/\eta} + F_E]\} \leq 0 \quad (14)$$

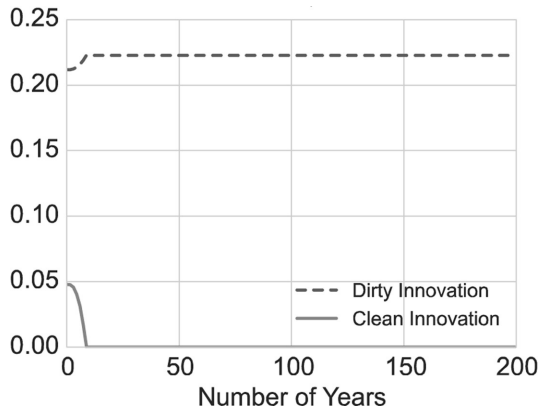
with equality if $E_t^j > 0$ (the mass of entrants fro technology j)

- Entrants will direct their R&D to technology j if $\bar{v}_t^j / (1 - s_t^j) > \bar{v}_t^{-j} / (1 - s_t^{-j})$.

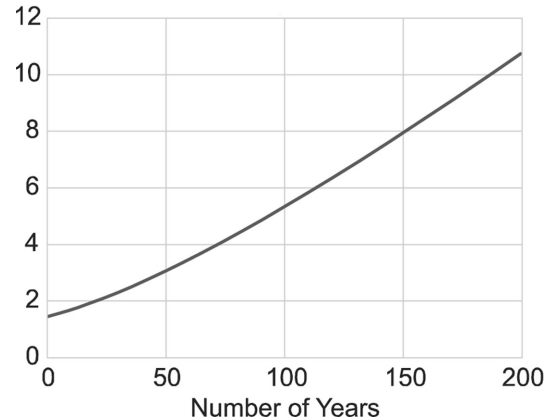
Calibration

- Three methods for calibration:
 - ① **Choose from external resources**: parameters related to environmental dynamics and resource extraction;
 - ② **Estimate from micro-data**: parameters related to innovation and R&D including L^s , α , η and the initial distribution of technology gaps;
 - ③ **Simulated method of moments**: other parameters θ , λ , F_I , F_E , R_0 and ζ .

Laissez-Faire Economy

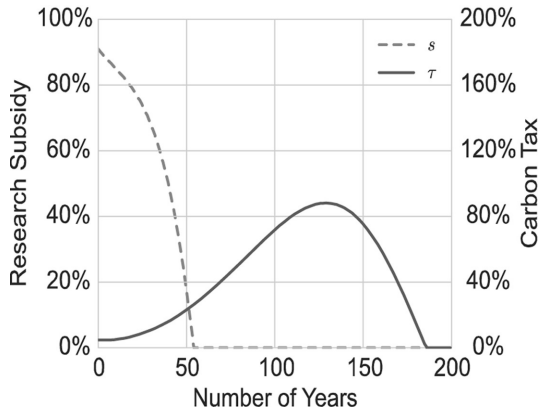


(a) Innovation Rates

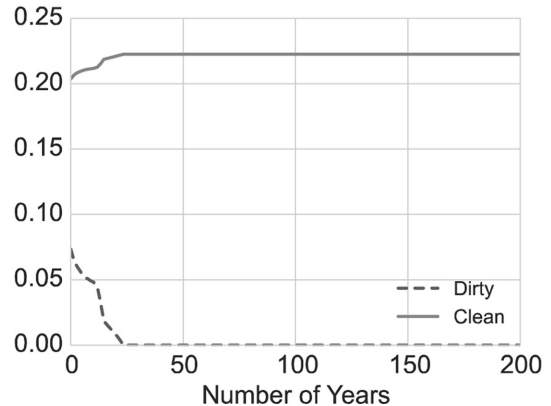


(b) Temperature Increase

Optimal Policies



(a) Optimal Policies



(b) Innovation Rates

Optimal Policies

- **Research subsidy is very high during the first few decades.** Social planner uses research subsidy to direct R&D from carbon-intensive dirty technologies towards clean technologies as soon as possible. Using carbon tax is socially cost because the initially technology gap is larger. → Large distortion from misallocation.
- **Carbon tax is hump shaped.** Carbon tax is used to correct externality. Larger technological gaps, larger distortion to direct production. As most production switches to the clean technology, there is less need for the carbon tax, so it is phased out.

Counterfactual Analysis

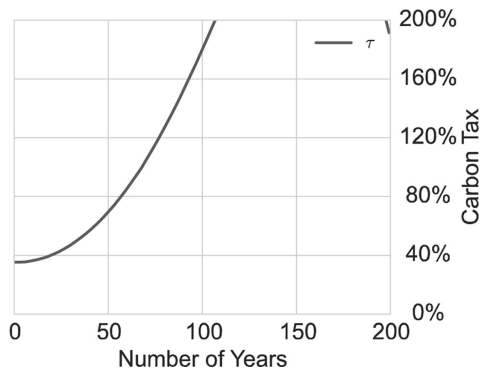


Figure 3: Optimal Carbon Tax

- The choice of an optimal policy relying only on a carbon tax.
- The carbon tax becomes much higher because it has two roles.
- The welfare cost of just relying on a carbon tax for optimal policy is 1.9 percent.

Counterfactual Analysis

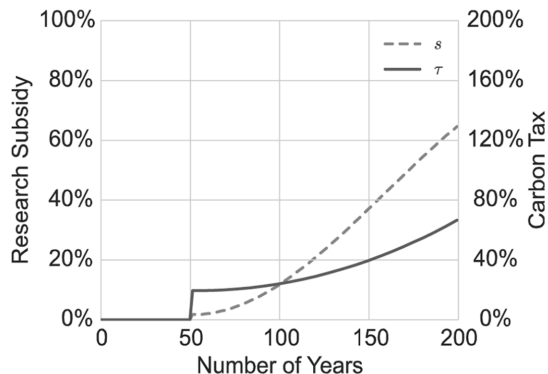


Figure 4: Optimal Policy Following a 50-year Delay

- Decaying the start of optimal policies by 50 years leads to less aggressive policies from that point onward.
- The intervening interval has generated a bigger technology gap between the clean and dirty sectors;
- The welfare loss 50-year delay is 1.7 percent. (depend on the parameter γ)

Conclusion

- Only carbon tax is not enough. Optimal policy relies heavily on research subsidies.

References

Acemoglu, D., Akcigit, U., Hanley, D., and Kerr, W. (2016). Transition to clean technology.
Journal of political economy, 124(1):52–104.